Leaflet for parents

Calculation

+ − x ÷

Vocabulary

Addition and Subtraction

add, addition, more, plus, increase
sum, total, altogether
score double, near double
how many more to make…?
subtract, subtraction, take (away), minus, decrease
leave, how many are left/left over?
difference between
half, halve
how many more/fewer is… than…?
how much more/less is…?
equals, sign, is the same as
tens boundary, hundreds boundary
units boundary, tenths boundary
inverse

Multiplication and Division

lots of, groups of
times, multiply, multiplication, multiplied by
multiple of, product
once, twice, three times… ten times…
times as (big, long, wide… and so on)
repeated addition
array, row, column
double, halve
share, share equally
one each, two each, three each…
group in pairs, threes… tens
equal groups of
divide, division, divided by, divided into
remainder
factor, quotient, divisible by
inverse

This booklet has been created to help ensure that children:

* will develop good mental maths skills;
* will have a good **understanding** of the four operations: + - x ÷;
* will be taught **consistently** throughout the school to use efficient and reliable written methods for each operation;
* have written methods that will support them when they are unable to carry out a calculation mentally;
* will use a calculator effectively, using their mental skills to monitor the process. They will check the steps involved and decide if the numbers displayed make sense.

At Jessie Younghusband School, children are introduced to the processes of calculation through practical exploration and investigation.

Mathematics is made fun and relevant through the use of story and problem-solving to provide a real life context. Children are taught to decide and identify when mental methods can be used and when written methods are needed to support mental procedures.

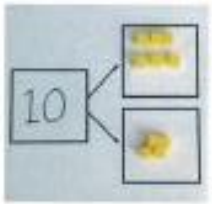
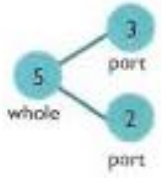
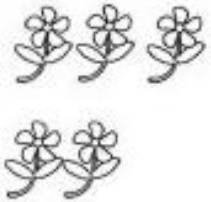
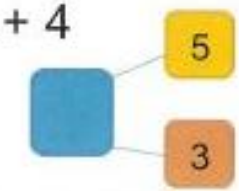
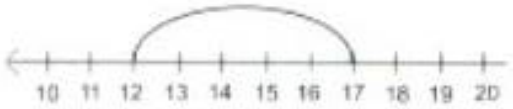
This booklet outlines the main methods taught at Jessie Younghusband School to support the progression from mental to written methods for calculation.

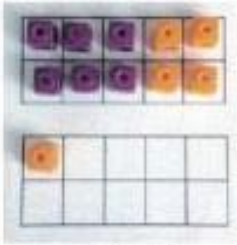
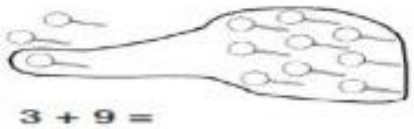
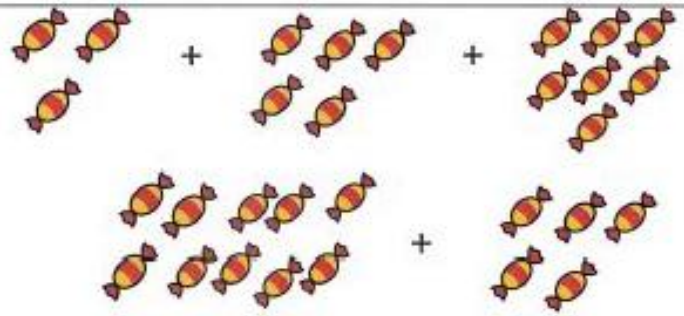
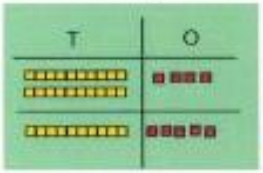
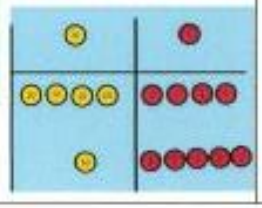
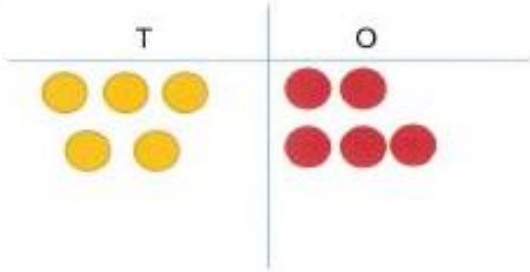
What sort of thing would it also help the children to know?

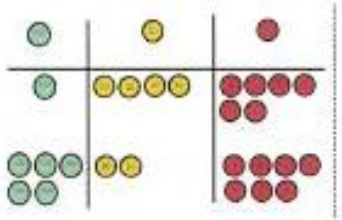
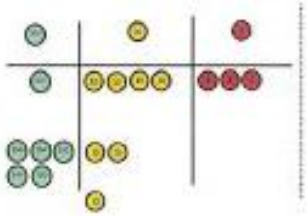
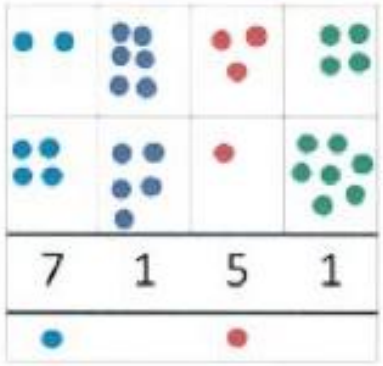
- Awareness of the commutative law – I don't know 5 x 7 but I do know 7 x 5 – the answer will be the same.
- Nearby facts – I don't know 8 x 5, but can do 10 x 5 – 10.
- If they don't know how to multiply by 4 and 8, they can scale answers up by repeated doubling.
- If they can't multiply by 5, they can x10 and then halve it.
- Partition where appropriate – e.g. 14 x 5 by doing 10 x 5 and then adding 4 x 5.
- Developing an awareness of whether an answer should be odd or even, as this will allow the chance to spot an unexpected mistake (for example if you add two odd numbers you will always get an even answer).

Progression in Calculations

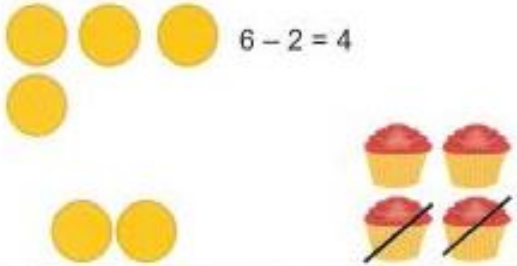
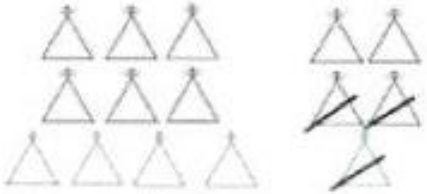
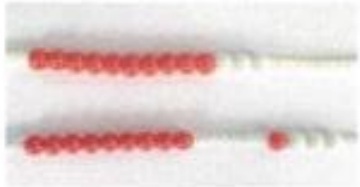
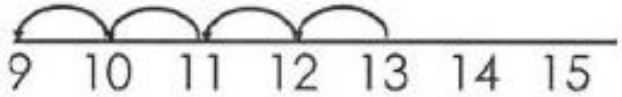
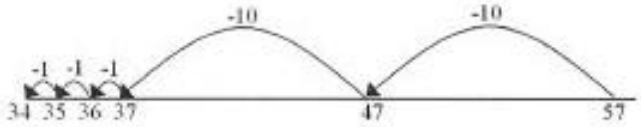
Addition

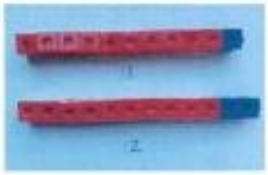
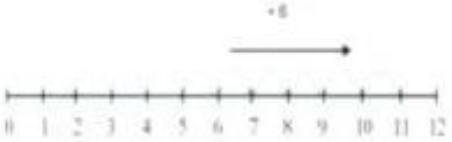
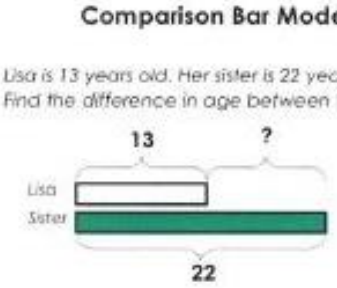
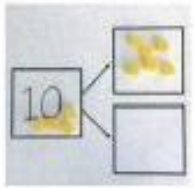
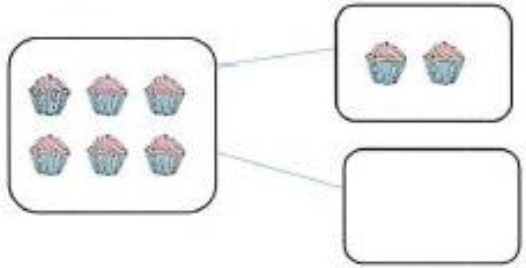
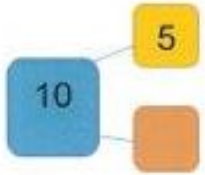
Objective and Strategies	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part-whole model	   Use cubes to add two numbers together as a group or in a bar. 	   Use pictures to add two numbers together as a group or in a bar. 	4 + 3 = 7 10= 6 + 4  Use the part-part whole diagram as shown above to move into the abstract.
Starting at the bigger number and counting on	 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	12 + 5 = 17  Start at the larger number on the number line and count on in ones or in one jump to find the answer.	5 + 12 = 17 Place the larger number in your head and count on the smaller number to find your answer.

Regrouping to make 10.	 6 + 5 = 11  Start with the bigger number and use the smaller number to make 10.	 3 + 9 = Use pictures or a number line. Regroup or partition the smaller number to make 10. 9 + 5 = 14 1 4 +1 +4 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20	7 + 4= 11 If I am at seven, how many more do I need to make 10. How many more do I add on now?
Adding three single digits	4 + 7 + 6= 17 Put 4 and 6 together to make 10. Add on 7.  Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.	 Add together three groups of objects. Draw a picture to recombine the groups to make 10.	4 + 7 + 6 = 10 + 7 10 = 17 Combine the two numbers that make 10 and then add on the remainder.
Column method- no regrouping	24 + 15= Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters.  	After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions. 	Calculations 21 + 42 = 21 + 42

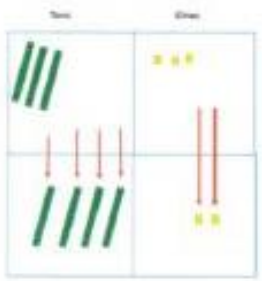
Column method- regrouping	Make both numbers on a place value grid.  146 + 527 Add up the units and exchange 10 ones for one 10.  146 + 527 Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added. This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100. As children move on to decimals, money and decimal place value counters can be used to support learning.	Children can draw a pictoral representation of the columns and place value counters to further support their learning and understanding.  7 1 5 1	Start by partitioning the numbers before moving on to clearly show the exchange below the addition. 20 + 5 40 + 8 60 + 13 = 73 As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here. 72.8 + 54.6 127.4 1 1 £23.59 + £7.55 £31.14 23.361 9.080 59.770 + 1.300 93.511 2 1 2

Subtraction

Objective and Strategies	Concrete	Pictorial	Abstract
Taking away ones	Use physical objects, counters, cubes etc to show how objects can be taken away. 	Cross out drawn objects to show what has been taken away.  15 – 3 = 12	18 -3= 15 8 – 2 = 6
Counting back	Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones.  Use counters and move them away from the group as you take them away counting backwards as you go. 	Count back on a number line or number track  Start at the bigger number and count back the smaller number showing the jumps on the number line.  This can progress all the way to counting back using two 2 digit numbers.	Put 13 in your head, count back 4. What number are you at? Use your fingers to help.

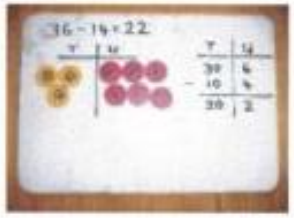
<table>
<tr>
<td>Find the difference</td>
<td>Compare amounts and objects to find the difference.
 
Use cubes to build towers or make bars to find the difference
 
Use basic bar models with items to find the difference</td>
<td> 
Count on to find the difference.
Draw bars to find the difference between 2 numbers.
  </td>
<td>Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches.</td>
</tr>
<tr>
<td>Part Part Whole Model</td>
<td> 
Link to addition- use the part whole model to help explain the inverse between addition and subtraction.
If 10 is the whole and 6 is one of the parts. What is the other part?
10 - 6 =</td>
<td>Use a pictorial representation of objects to show the part part whole model.
  </td>
<td> 
Move to using numbers within the part whole model.</td>
</tr>
<tr>
<td>Make 10</td>
<td>14 – 9 =
 
Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9.</td>
<td> 
Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.</td>
<td>16 – 8=
How many do we take off to reach the next 10?
How many do we have left to take off?</td>
</tr>
</table>

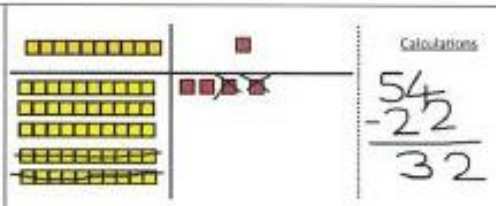
Column method without regrouping



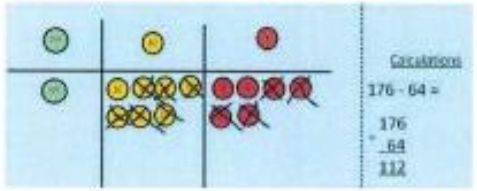
Use Base 10 to make the bigger number then take the smaller number away.

Show how you partition numbers to subtract. Again make the larger number first.





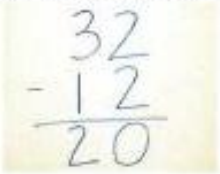
Draw the Base 10 or place value counters alongside the written calculation to help to show working.



47 - 24 = 23

40 + 7
- 20 + 4
20 + 3

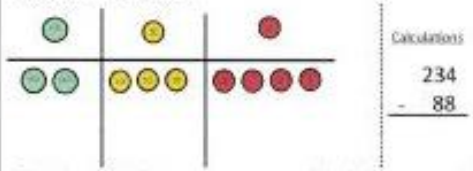
This will lead to a clear written column subtraction.



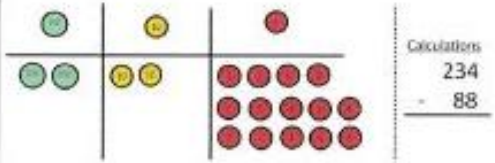
Column method with regrouping

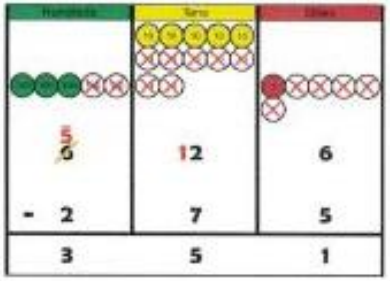
Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.

Make the larger number with the place value counters



Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.





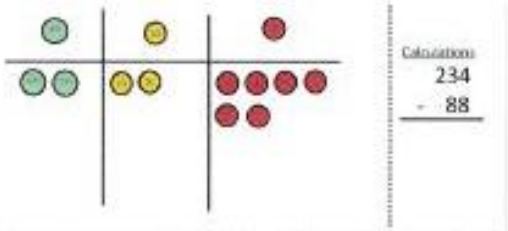
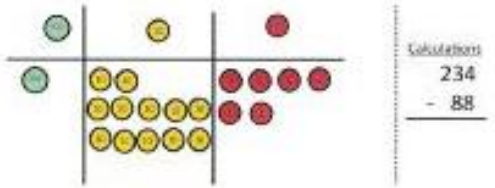
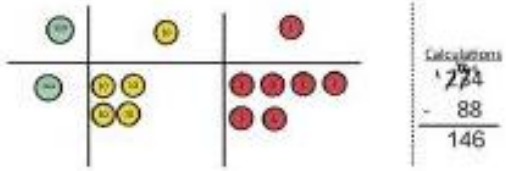
Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.

When confident, children can find their own way to record the exchange/regrouping.

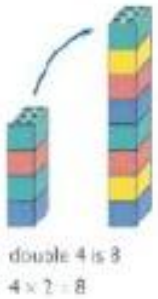
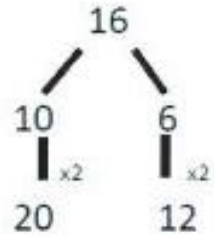
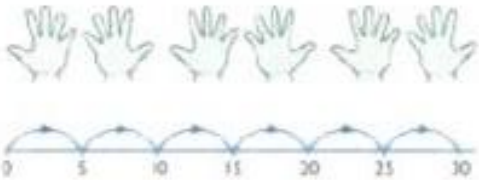
Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/regroup.

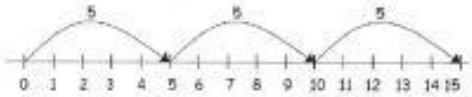
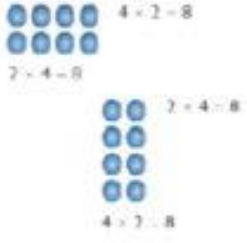
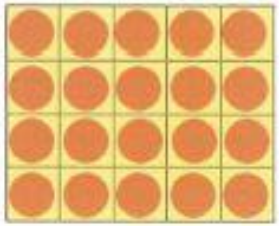
Children can start their formal written method by partitioning the number into clear place value columns.

Moving forward the children use a more compact method.

	Now I can subtract my ones.  Calculations 234 - 88 Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.  Calculations 234 - 88 Now I can take away eight tens and complete my subtraction  Calculations 234 - 88 146 Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.		This will lead to an understanding of subtracting any number including decimals. 5 12 1 2 6 3 . 0 - 2 6 . 5 2 3 6 . 5

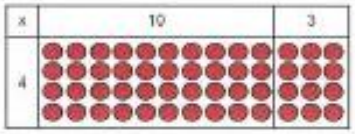
Multiplication

Objective and Strategies	Concrete	Pictorial	Abstract
Doubling	Use practical activities to show how to double a number.  	Draw pictures to show how to double a number. Double 4 is 8 	 Partition a number and then double each part before recombining it back together.
Counting in multiples	  Count in multiples supported by concrete objects in equal groups.	 Use a number line or pictures to continue support in counting in multiples.	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30

Repeated addition	   Use different objects to add equal groups.	There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?  2 add 2 add 2 equals 6  5 + 5 + 5 = 15	Write addition sentences to describe objects and pictures. 
Arrays- showing commutative multiplication	Create arrays using counters/ cubes to show multiplication sentences.  	Draw arrays in different rotations to find **commutative** multiplication sentences.   Link arrays to area of rectangles.	Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15

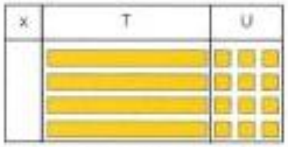
Grid Method

Show the link with arrays to first introduce the grid method.



4 rows of 10
4 rows of 3

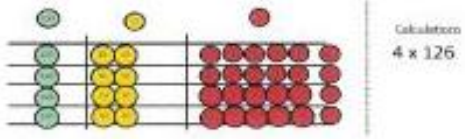
Move on to using Base 10 to move towards a more compact method.



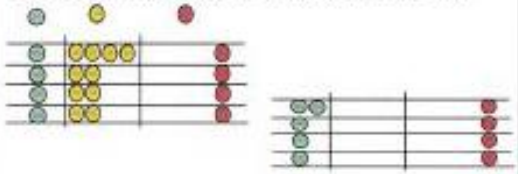
4 rows of 13

Move on to place value counters to show how we are finding groups of a number.We are multiplying by 4 so we need 4 rows.

Fill each row with 126.



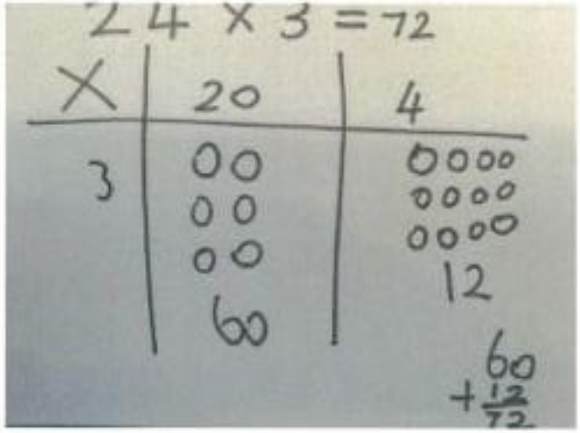
Add up each column, starting with the ones making any exchanges needed.



Then you have your answer.

Children can represent the work they have done with place value counters in a way that they understand.

They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.



Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

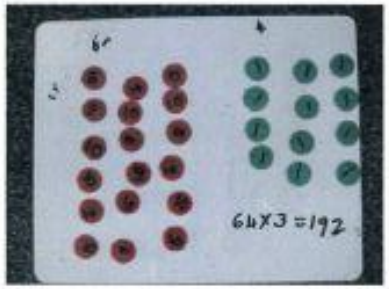
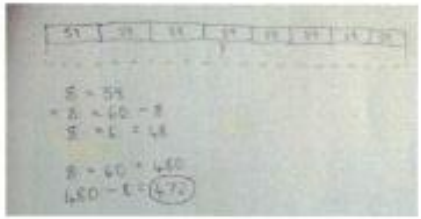
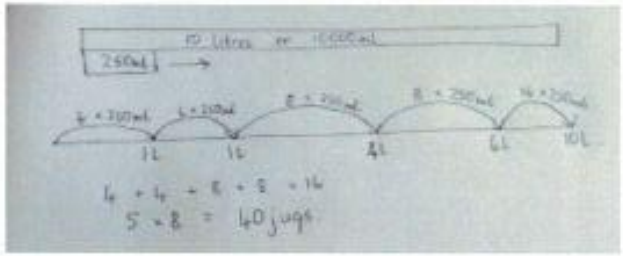
×	30	5
7	210	35

210 + 35 = 245

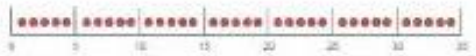
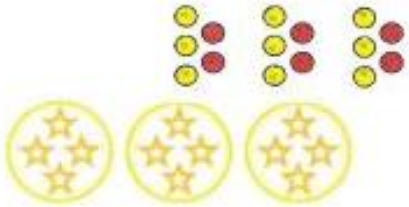
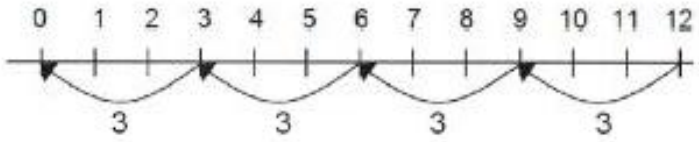
Moving forward, multiply by a 2 digit number showing the different rows within the grid method.

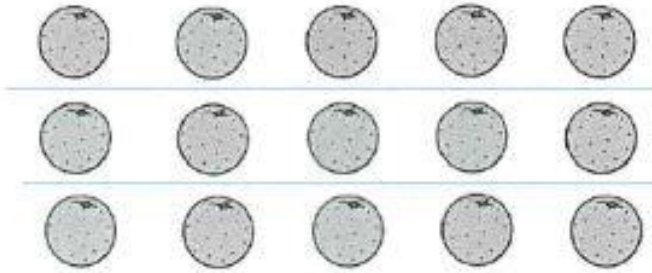
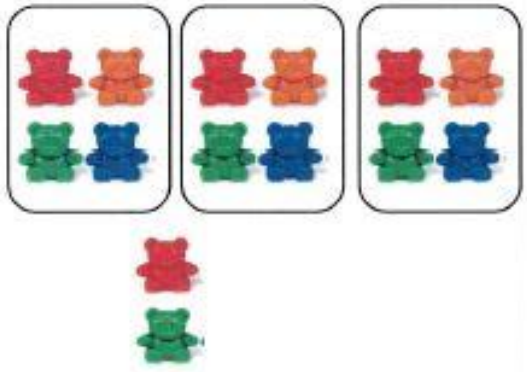
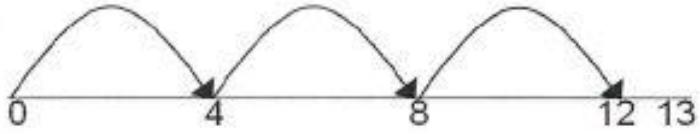
	10	8
10	100	80
3	30	24

X	1000	300	40	2
10	10000	3000	400	20
8	8000	2400	320	16

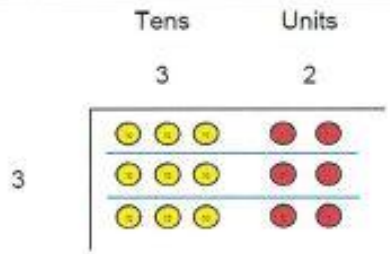
Column multiplication	Children can continue to be supported by place value counters at the stage of multiplication.  It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.	Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.  	Start with long multiplication, reminding the children about lining up their numbers clearly in columns. If it helps, children can write out what they are solving next to their answer. 32 x 24 8 (4 x 2) 120 (4 x 30) 40 (20 x 2) 600 (20 x 30) 768 This moves to the more compact method. 1342 x 18 13420 10736 24156

Division

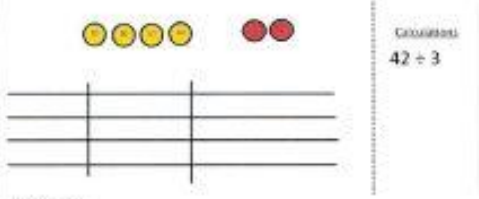
Objective and Strategies	Concrete	Pictorial	Abstract
Sharing objects into groups	   I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.  $8 \div 2 = 4$	Share 9 buns between three people. $9 \div 3 = 3$
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.   $96 \div 3 = 32$ 	Use a number line to show jumps in groups. The number of jumps equals the number of groups.  Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group. 	$28 \div 7 = 4$ Divide 28 into 7 groups. How many are in each group?

Division within arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	 Draw an array and use lines to split the array into groups to make multiplication and division sentences.	Find the inverse of multiplication and division sentences by creating four linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7
Division with a remainder	14 ÷ 3 = Divide objects between groups and see how much is left over 	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount and clearly show a remainder. 	Complete written divisions and show the remainder using r. 

Short division

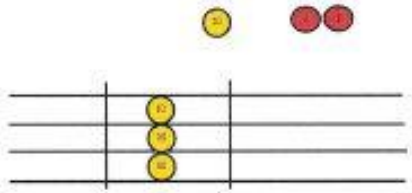


Use place value counters to divide using the bus stop method alongside

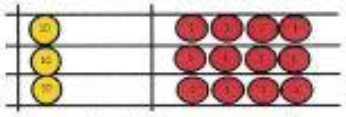


42 ÷ 3=

Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.

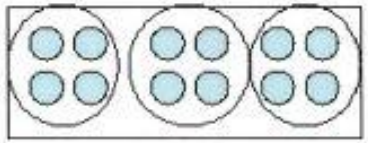


We exchange this ten for ten ones and then share the ones equally among the groups.



We look how much in 1 group so the answer is 14.

Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.



Encourage them to move towards counting in multiples to divide more efficiently.

Begin with divisions that divide equally with no remainder.

$$\begin{array}{r} 218 \\ 4\overline{)872} \\ \underline{3} \\ \end{array}$$

Move onto divisions with a remainder.

$$\begin{array}{r} 86\ r\ 2 \\ 5\overline{)432} \\ \underline{3} \\ \end{array}$$

Finally move into decimal places to divide the total accurately.

$$\begin{array}{r} 14.6 \\ 35\overline{)511.0} \\ \underline{16} \\ \quad 21 \end{array}$$

Objectives from the National Curriculum that involve mental maths: recognising and using patterns and developing efficient mental strategies.

Year	Counting / Place Value	Addition and subtraction / multiplication and division
R	Recognise numeral 1-5, 1-10, 1-20 Count up to 4, 10, 20 objects from a larger group Count actions or objects that can't be moved Select the correct numeral to represent 1-5, 1-10, 1-20 objects Count an irregular arrangement of up to 5, 10, 20 objects Place numbers 1-5, 1-10, 1-20 in order Know one more / one less that a number 1-5, 1-10, 1-20 objects Estimate numbers in a group	Add and subtract two single digit numbers counting on or back to find the answer Estimate numbers in a group
1	Count to and across 100, forward and backward, from any number Identify one more and less than a given number Count in multiples of 2, 5 and 10 Recognise place value in numbers beyond 20 by reading, writing, counting and comparing numbers up to 100 Read and write numbers to 100	Memorise and reason with number bonds to 10 and 20 e.g. 9+7=16, 16-7=9 Add and subtract one-digit and two-digit numbers to 20, including 0 Count in multiples of 2, 5 and 10 (develop patterns, like odd and even) Grouping and sharing small quantities to understand multiplication and division (doubling, simple fractions)
2	Count in steps of 2, 3, 5 and 10 from any number, forward and backward Recognise the value of each digit in a two-digit number (T O) Estimate larger numbers using different representations, including number line Compare and order numbers from 0 to 100 using <, >, = Partition numbers in different ways: 23 = 20 + 3 and 10 + 13 Read and write numbers to at least 100	Recall and use multiplication and division facts for the 2, 5 and 10 x tables = make connection between each Recognise odd and even numbers Recall and use addition and subtraction facts to 20 fluently and derive and use related facts to 100 Add and subtract numbers, using concrete, pictorial representations and mentally (2-digit and ones, 2-digit and tens, 2 two-digit, 3 1-digit numbers) Recognise and use inverse relationships

Objectives from the National Curriculum that involve mental maths: recognising and using patterns and developing efficient mental strategies.

Year	Counting / Place Value	Addition and subtraction / multiplication and division
3	Count in multiples of 4, 8, 50 and 100 Find 100 more / less than a number Recognise the place value of each digit in a 3-digit number, applying partitioning: 146 = 100 + 46 and 130 + 16 Compare and order numbers up to 1000 Count in ones, tens and hundreds up to 1000 Count up and down in tenths	Add and subtract numbers mentally including 3-digit numbers and ones, tens and hundreds Recall and use multiplication and division facts for the 3, 4 and 8 x tables Doubling to connect the 2, 4 and 8 x tables Develop efficient mental methods using commutativity and associativity (4 x 12 x 5 = 4 x 5 x 12 = 20 x 12 = 240)
4	Count in multiples of 6, 7, 25 and 1000 Find 1000 more or less than a given number Count backwards through zero to include negative numbers Recognise the place value of each digit in a 4-digit number Order and compare numbers beyond 1000 Round any number to the nearest 10, 100 or 1000 Count up and down in hundredths	Practise mental methods for addition and subtraction with increasingly larger numbers Recall multiplication and division facts for tables up to 12 x Use place value, known and derived facts to multiply and divide mentally Recognise and use factor pairs and commutativity in mental calculations
5	Read, write, order and compare numbers to at least 1 000 000 and determine the value of each digit Count forwards and backwards in steps of powers of 10 from any number Count forward and backward with positive and negative whole numbers Round any number to the nearest 10, 100, 1000, 10 000 and 1 000 000 Count up and down in simple fractions and decimals	Add and subtract numbers mentally with increasingly larger numbers 12 462 – 2 300= 10 162 Commit table facts to memory and use them confidently (factors and multiples) Recall prime numbers up to 19 and establish whether other numbers up to 100 are prime Multiply and divide numbers mentally drawing upon known facts, and those involving decimals, by 10, 100 and 100
6	Read, write, order and compare numbers up to 10 000 000 and determine the value of each digit Identify the value of each digit in numbers given to three decimal places	Perform mental calculations, including mixed operations and increasingly large numbers Continue to use all the multiplication tables Multiply and divide numbers, including decimals by 10, 100 and 1000

The Use of Calculators

Specific objectives for the use of a calculator span Years 4, 5 and 6, but in the context of exploring numbers and the number system, calculators are used at Jessie Younghusband School with children in all age groups across the Foundation Stage, Key Stage 1 and Key Stage 2.

Basic calculator skills include recognising numbers and symbols, learning how to use a calculator and recognising when it is appropriate to do so. Later, calculators support the teaching of mathematics where the aim is to focus on solving a problem rather than on the process of calculation.

Using a Calculator Vocabulary

calculator, display, key
enter, clear, sign
change constant, recurring, memory, operation key